Liquid Welfare Guarantees for Learning in Sequential Budgeted Auctions

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Introduction

Autobidders

Algorithms for online auctions

90% of ad dollars transacted using autobidders, over $123 billion in US, 2022

Fast-changing environment, hard budget limits

Player Assumptions

T rounds/items

n players

Player i's **value** in round t: $v_{it} \in [0,1]$

Additive Valuations: If player i wins rounds S_i, total value is $V_i = \sum_{t \in S_i} v_{it}$

Budgeted quasi-linear utilities: Budget B_i and payment P_i then utility

$$U_i = \begin{cases} V_i - P_i, & \text{if } P_i \le B_i \\ -\infty, & \text{otherwise} \end{cases}$$

Liquid Welfare

Generalization of social welfare for budget-limited players

Player i has liquid welfare

$$\mathrm{LW}_i = \min\{V_i, B_i\}$$

Total liquid welfare is $\mathrm{LW} = \sum_i \mathrm{LW}_i$ and optimal is LW^*

Shading Multipliers to bid

Control spending when budget constrained

Shade value to bid λv_{it} for some $\lambda \in [0,1]$

Balseiro and Gur 2017: iteratively adapt shading multiplier for individual utility guarantees in second-price, e.g. no-regret

Gaitonde et al. 2023: above algorithm by all players implies $\mathrm{LW} \ge \frac{1}{2}\mathrm{LW}^*$ (for iid player values)

Behavioral Assumption

Player i has **competitive ratio** $\gamma \ge 1$ and **regret** Reg if competitive with best multiplier in hindsight:

$$U_i \ge \frac{\sup_{\lambda \in [0,1]} \hat{U}_i(\lambda) - \mathrm{Reg}}{\gamma}$$

$\hat{U}_i(\lambda)$: player i's utility if she used multiplier λ every round, i.e. bid λv_{it} until out of budget

(Lack of) Guarantees in Second-price Auctions

Even if

- $n = 2$
- $\gamma = 1$
- $\mathrm{Reg} = 0$
- constant player values

it can hold $\frac{\mathrm{LW}}{\mathrm{LW}^*} = 0$

Welfare Guarantees in First-price

First-price Auctions

If every player has competitive ratio at most γ and regret Reg, then

$$\mathrm{LW} \ge \frac{\mathrm{LW}^* - O(n)\mathrm{Reg}}{\gamma + \frac{1}{2} + O\left(\frac{1}{\gamma}\right)}$$

Denominator becomes 2.41 when $\gamma = 1$

- Player values can be **adversarial**
- **Any player algorithms** with the behavioral assumption

More general result than previous work

First-price Upper Bounds

For any $\gamma \ge 1$ if

- $n = 2$
- $\mathrm{Reg} = 0$
- constant player values

it can hold that $\mathrm{LW} \le \frac{1}{\max\{\gamma, 2\}}\mathrm{LW}^*$

Submodular valuations

If players have submodular valuations across rounds then

$$\mathrm{LW} \ge \frac{\mathrm{LW}^* - O(n)\mathrm{Reg}}{\gamma + 1 + O\left(\frac{1}{\gamma}\right)}$$

Denominator becomes 2.62 when $\gamma = 1$

Algorithmic Results

Player i with additive valuation can guarantee with high probability

$$U_i \ge \frac{\sup_{\lambda \in [0,1]} \hat{U}_i(\lambda) - \tilde{O}\left(T^{3/2}/B_i\right)}{T/B_i}$$

for adversarial player values and bids

Meaningful guarantee if $B_i \ge T^{1/2+\Omega(1)}$

Conclusion

Weak individual player guarantees imply aggregate welfare in first-price, even for adversarial player values

In high contrast to second-price where no such guarantees hold

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