Robust Pseudo-Markets for Reusable Public Resources

EC 2023

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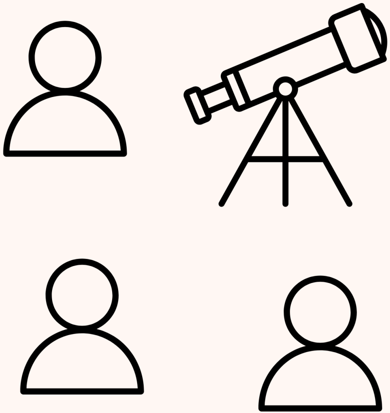
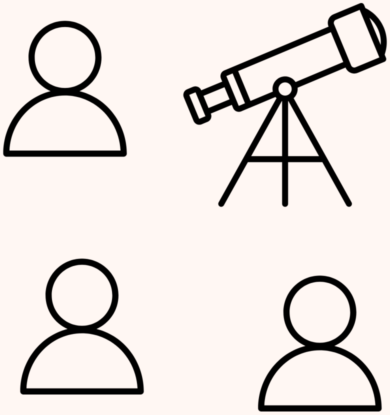
- n agents



Reusable Resource sharing

- n agents
- T rounds



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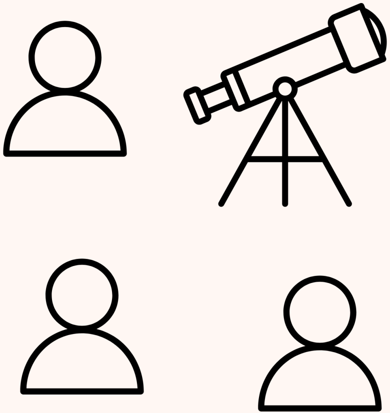

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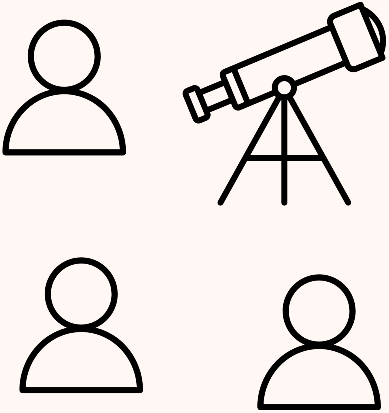

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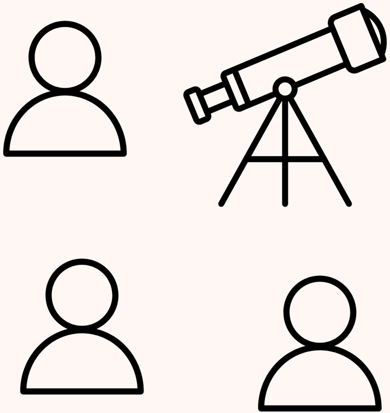

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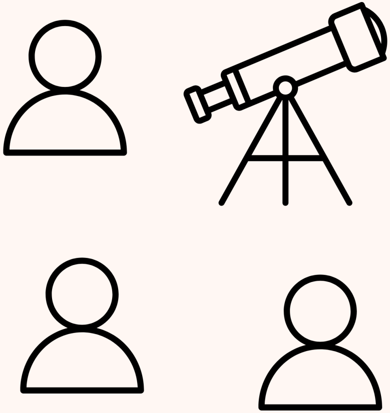
  - Telescope
  - Gene sequencer
  - Computing cluster



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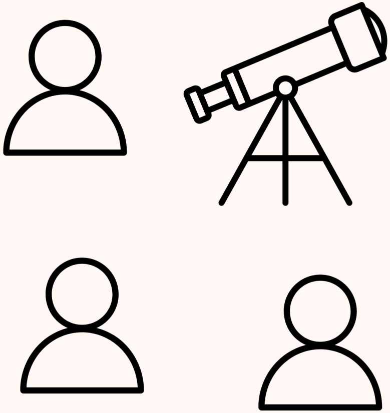
- Simulate market with artificial currency



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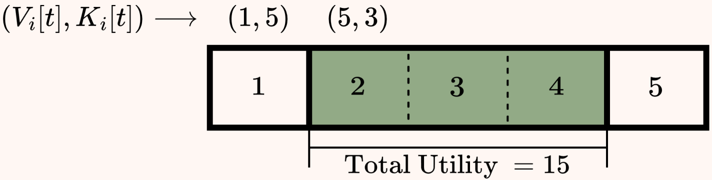

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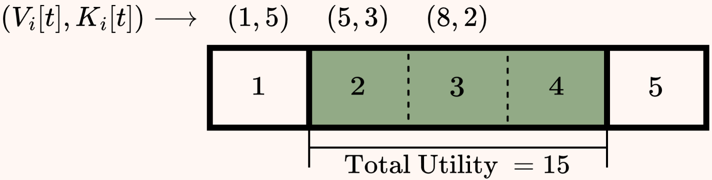

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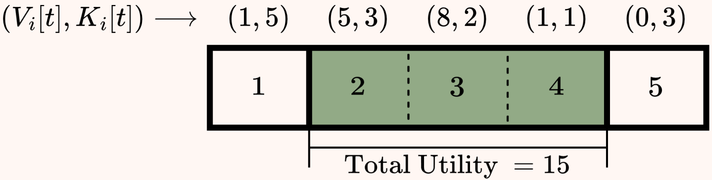

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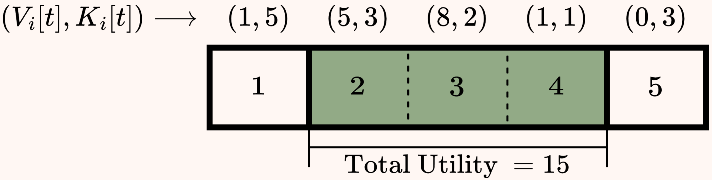

- Bayesian setting: $(V_i[t], K_i[t]) \sim F_i$

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- For single round demands where

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  can hope for total utility $\approx \alpha_i T$

Ideal Utility

Individual agent guarantee

Defined in [Gorokh-Banerjee-Iyer, EC'21] for single round demands, related to [Kalai-Smorodinsky, Econometrica'75]

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Theorem – Ideal Utility Calculation

$v_i^\star$ and $\pi_i^\star$ can be computed by an LP.

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   - Collect desired durations and *per-round* bids
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   - Multi-round bids must be at least reserve r

Ideal Utility Guarantees

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Theorem – Robust Guarantee

If $r \geq 1$ then even under adversarial competition agent i can guarantee expected utility

$$v_i^\star T \min\left\{\frac{1}{r}, 1 - \frac{1-\alpha_i}{r}\right\} - O\left(\sqrt{T}\right)$$

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Maximized if $r = 2$:

$$\frac{v_i^\star}{2} T - O\left(\sqrt{T}\right)$$

Guarantee Intuition

- If $r = 1$ others block agent i

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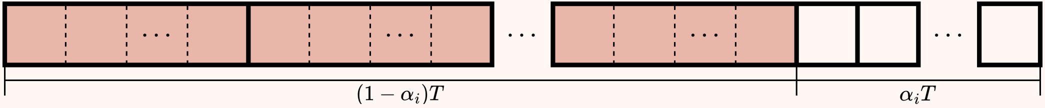

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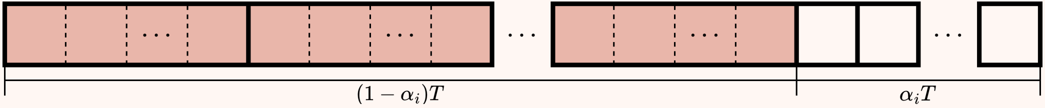
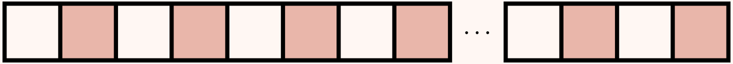


- If $r = 2$ others win at most $\approx \frac{T}{2}$ rounds



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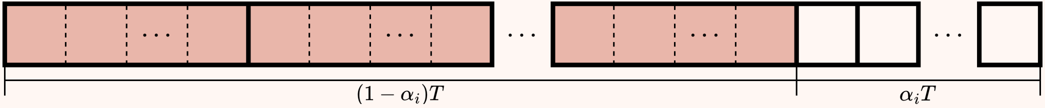

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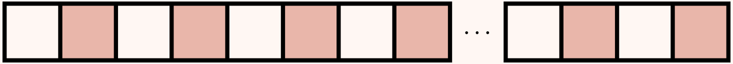

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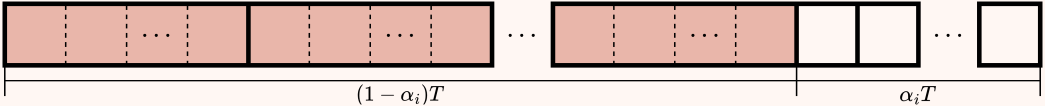

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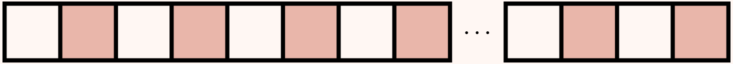

  - If $K_i[t] = 1$ agent i wins α_i fraction of free rounds
  - If $K_i[t] = 2$ rely on martingale argument

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- 2 bound on the PoA

Theorem – Optimality of mechanism

No mechanism can guarantee every agent i expected utility more than

$$v_i^\star T\left(\frac{1}{2} + O\left(\frac{1}{k_{\max}}\right)\right)$$

as $n \to \infty$.

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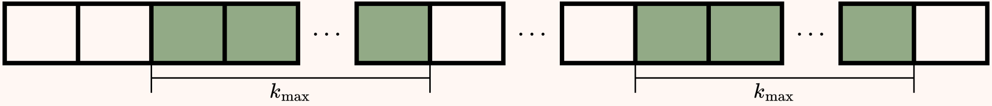

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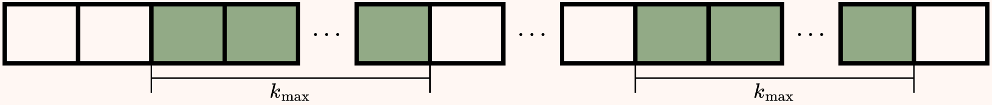

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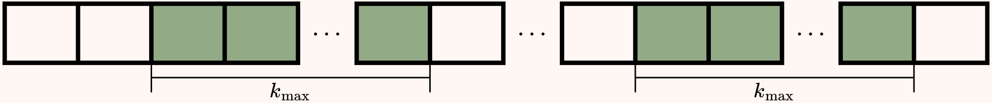
- Social welfare at most $\frac{T}{2}$



Summary

- Public reusable resource sharing
- Ideal utility: individual agent benchmark
- First-Price Pseudo-Auction with Multi-Round Reserves
- Robust Bidding Policy: guarantees half of total ideal utility
- No mechanism guarantees everyone more than half of total ideal utility